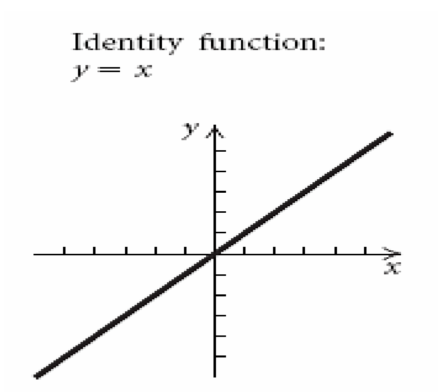
Transformations of Functions

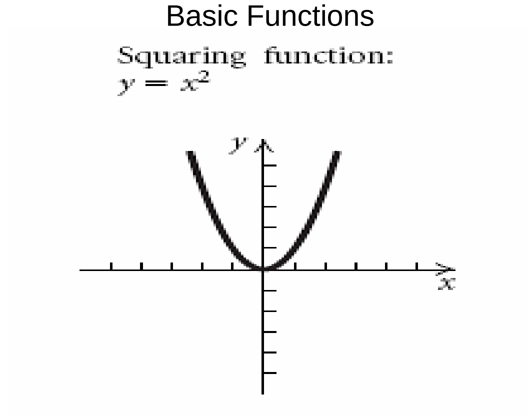
Objectives

- Recognize graphs of common functions
- Use vertical shifts to graph functions
- Use horizontal shifts to graph functions
- Use reflections to graph functions
- Use vertical stretching & shrinking to graph functions
- Use horizontal stretching & shrinking to graph functions
- Graph functions w/ sequence of transformations

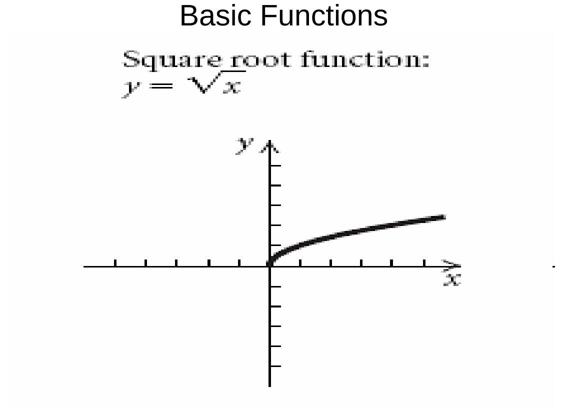
Basic Functions



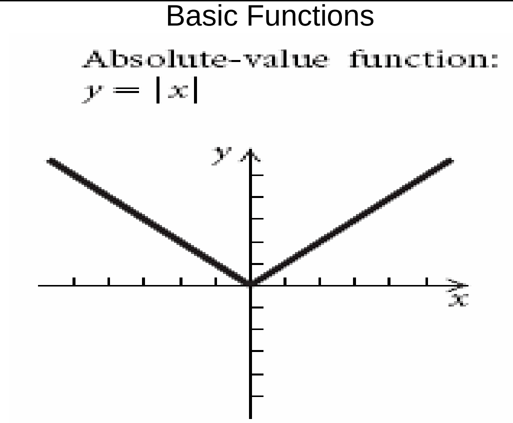
Basic Functions



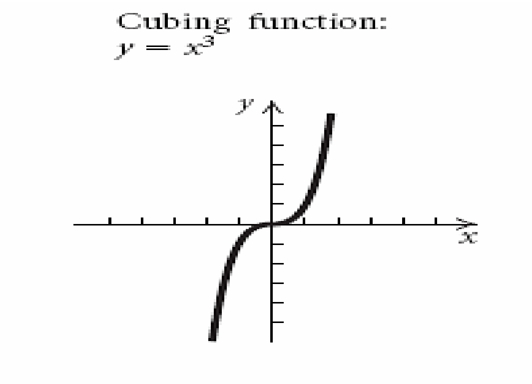
Basic Functions



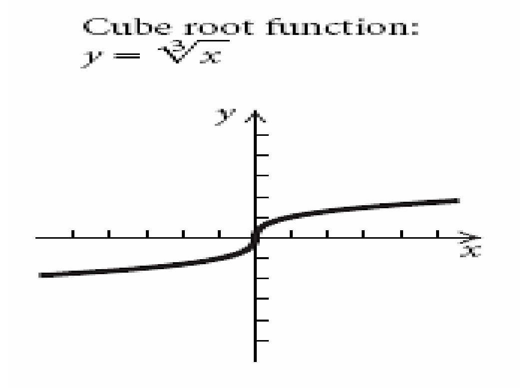
Basic Functions



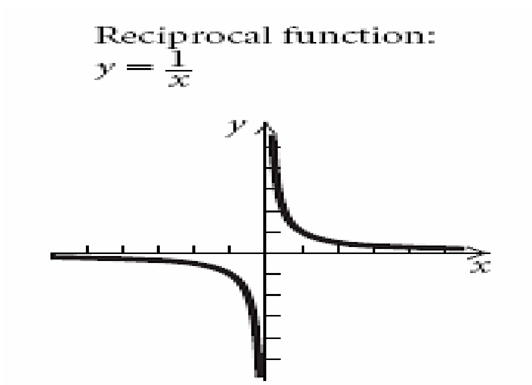
Basic Functions



Basic Functions



Basic Functions



Transformations with the Squaring Function

$f(x) = x^2$

$g(x) = x^2+3$

$h(x) = x^2-2$

$q(x) = (x-2)^2$

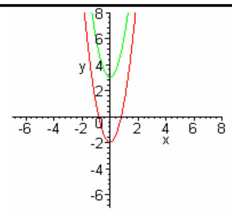
$r(x) = (x+4)^2$

$s(x) = -x^2$

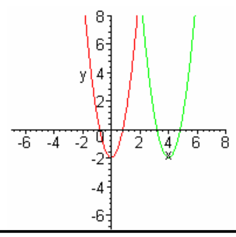
$t(x) = 3 \cdot x^2$

$v(x) = \left(\frac{1}{2}\right) \cdot x^2$

- **Vertical shifts**
  - Moves the graph up or down
  - Impacts only the “y” values of the function
  - No changes are made to the “x” values

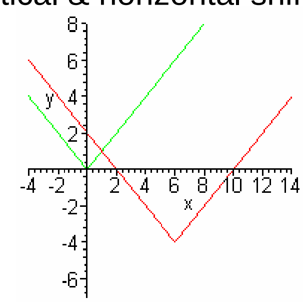


- **Horizontal shifts**
  - Moves the graph left or right
  - Impacts only the “x” values of the function
  - No changes are made to the “y” values



Combining a vertical & horizontal shift

- Example of function that is shifted down 4 units and right 6 units from the original function.



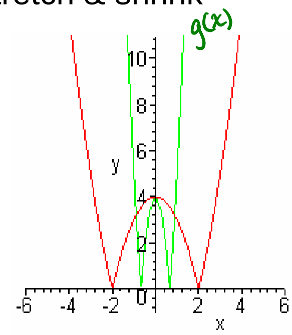
$$f(x) = |x|, g(x) = |x-6| - 4$$

Reflecting

- Across x-axis (y becomes negative, -f(x))
- Across y-axis (x becomes negative, f(-x))

Horizontal stretch & shrink

- We're MULTIPLYING by an integer (not 1 or 0).
- x's do the opposite of what we think they should. (If you see 3x in the equation where it used to be an x, you DIVIDE all x's by 3, thus it's compressed horizontally.)

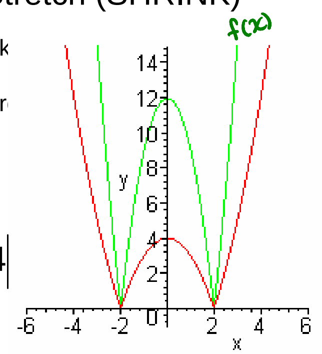


$$g(x) = \left|(3x)^2 - 4\right|$$

Vertical Stretch (SHRINK)

- y's do what we think they should: If you see 3(f(x)), all y's ar[e] MULTIPLIED by 3 (it's now 3 times as high or low!)

$$f(x) = 3\left|x^2 - 4\right|$$



Sequence of Transformations

- Follow order of operations.
- Select two points (or more) from the original function and move that point one step at a time.

Example

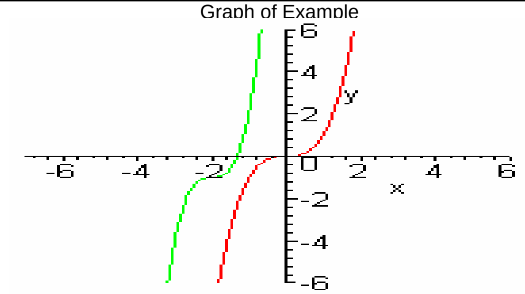
$$f(x) = x^3$$

$$3f(x+2) - 1 = 3(x+2)^3 - 1$$

f(x) contains **(-1,-1), (0,0), (1,1)**

- 1st transformation would be (x+2), which moves the function left 2 units. (subtract 2 from each x)
  Points are now (-3,-1), (-2,0), (-1,1)
- 2nd transformation would be 3 times all the y's.
  Points are now (-3,-1), (-2,0), (-1,3)
- 3rd transformation would be subtract 1 from all y's.
  Points are now **(-3,-2), (-2,-1), (-1,2)**

Graph of Example



$$f(x) = x^3$$

$$g(x) = 3f(x+2) - 1 = 3(x+2)^3 - 1$$

Transformation Rules

- **Equation** — **How to obtain the graph**
  For (c > 0)

Vertical Shifts

- y = f(x) + c — Shift graph y = f(x) up **c** units
- y = f(x) - c — Shift graph y = f(x) down **c** units

Horizontal Shifts

- y = f(x – c) — Shift graph y = f(x) right **c** units
- y = f(x + c) — Shift graph y = f(x) left **c** units

Transformation Rules

- **Equation** — **How to obtain the graph**

Reflection about the x-axis

- y = -f(x) (c > 0) — Reflect graph y = f(x) over x-axis
  Multiply f(x) by -1

Reflection about the y-axis

- y = f(-x) (c > 0) — Reflect graph y = f(x) over y-axis
  x is replaced with *–x*

Transformation Rules

- **Equation** — **How to obtain the graph**

Vertical stretching or shrinking

- y = c f(x) (c > 1) — Stretch graph y = f(x) vertically by factor of **c**
  Multiply y-coordinates of y = f(x) by c
- y = c f(x) (0 < c < 1) — Shrink graph y = f(x) vertically by factor of **c**
  Multiply y-coordinates of y = f(x) by c

Transformation Rules

- **Equation** — **How to obtain the graph**

Horizontal stretching or shrinking

- y = f(cx) (c > 1) — Shrink graph y = f(x) vertically by factor of **c**
  Divide x-coordinates of y = f(x) by c
- y = f(cx) (0 < c < 1) — Stretch graph y = f(x) vertically by factor of **c**
  Divide x-coordinates of y = f(x) by c